The present document summarizes those methods for constructing (approximate) confidence intervals for standardized effect sizes that were found to be most accurate in:

Viechtbauer, W. (in press). Approximate confidence intervals for standardized effect sizes in the two-independent and two-dependent samples design. *Journal of Educational and Behavioral Statistics*.

For more details, you will have to look at the entire article.

(1) Two-Independent Samples Design

In the two-independent samples design, subjects are (randomly) assigned to an experimental (E) and a control (C) group. Assume that the scores in the two groups are sampled from normal distributions with expectations μ_E and μ_C and common variance σ^2. The standardized mean difference effect size is then given by

$$\delta_2 = \frac{\mu_E - \mu_C}{\sigma}.$$

An estimate of δ_2 is given by

$$d_2 = \frac{\bar{X}_E - \bar{X}_C}{s_p},$$

where $\bar{X}_E$ and $\bar{X}_C$ denote the sample means of the n_E and n_C scores in the two groups,

$$s_p = \sqrt{\frac{s_E^2(n_E - 1) + s_C^2(n_C - 1)}{n_E + n_C - 2}},$$

and s_E^2 and s_C^2 are the observed variances of the scores in the respective groups. However, d_2 is a positively biased estimator of δ_2 (i.e., it is too large on average). An unbiased estimate of δ_2 can be obtained with

$$g_2 = c(m)\left(\frac{\bar{X}_E - \bar{X}_C}{s_p}\right),$$

where

$$c(m) = \frac{\Gamma\left(\frac{m}{2}\right)}{\sqrt{\frac{m}{2}}\,\Gamma\left(\frac{m-1}{2}\right)} \approx 1 - \frac{3}{4m - 1}$$

and $m = n_E + n_C - 2$. An approximate 95% confidence interval for δ_2 can then be obtained with

$$d_2 \pm 1.96 \times \sqrt{\frac{n_E + n_C}{n_E n_C} + \frac{d_2^2}{2m}}.$$

(2) Two-Dependent Samples Design

In the two-dependent samples design, a single group of n subjects is measured on two occasions, such as before and after receiving some treatment. Assume that the scores X_1 and X_2

obtained at time 1 and time 2 are sampled from normal distributions with expected values μ_1 and μ_2 and common variance σ^2. Now define the random variable $D = X_2 - X_1$. It follows that D is normally distributed with expected value $\mu_D = \mu_2 - \mu_1$ and variance $\sigma_D^2 = 2\sigma^2(1-\rho)$, where ρ is the correlation between the scores at time 1 and time 2. Two different standardized effect sizes have been discussed in the literature, namely

$$\delta_D = \frac{\mu_D}{\sigma_D} = \frac{\mu_2 - \mu_1}{\sigma\sqrt{2(1-\rho)}},$$

where the mean difference is standardized by the standard deviation of the differences scores, and

$$\delta_{D2} = \frac{\mu_D}{\sigma} = \frac{\mu_2 - \mu_1}{\sigma},$$

where the mean difference is standardized by the standard deviation of the raw scores.

(a) Confidence Interval for Parameter δ_D

An estimate of δ_D is given by

$$d_D = \frac{\bar{D}}{s_D},$$

where $\bar{D}$ and s_D^2 denote the mean and the variance of the D scores, respectively. An unbiased estimate of δ_D can be obtained with

$$g_D = c(m)\left(\frac{\bar{D}}{s_D}\right),$$

where $m = n - 1$. Finally, an approximate 95% confidence interval for δ_D can then be obtained with

$$d_D \pm 1.96 \times \sqrt{\frac{1}{n} + \frac{d_D^2}{2m}}.$$

(b) Confidence Interval for Parameter δ_{D2}

An estimate of δ_{D2} is given by

$$d_{D2} = \frac{\bar{D}}{s_1},$$

where s_1 is the pre-treatment standard deviation. An unbiased estimate is given by

$$g_{D2} = c(m)\left(\frac{\bar{D}}{s_1}\right),$$

where $m = n - 1$. Finally, an approximate 95% confidence interval for δ_D can then be obtained with either

$$d_{D2} \pm 1.96 \times \sqrt{\frac{2(1-r)}{n} + \frac{d_{D2}^2}{2n}}.$$

or

$$g_{D2} \pm 1.96 \times \sqrt{\frac{2(1-r)}{n} + \frac{g_{D2}^2}{2m}}.$$